EE512 - Applied Biomedical Signal Processing

Midterm Exam ("mock" exam which does NOT count for evaluation)

Remarks

- There are 4 exercises and 12 questions. The 4 exercises and 12 questions are worth 5/16 (approx. 0.3125) points each, hence a total of 5 points. There is an additional point for presence. **This information is for your own auto-evaluation.**
- The 4 exercises relate to the Lectures (Module 02 to Module 06).
- The 12 questions relate to the Modules 02 to 07 and corresponding Laboratories.
- All questions/problems are on the 3 pages**.**
- Focus on the essential ideas (like 5-10 lines for each question), but do not limit your answer to a couple of keywords.
- If you get stuck on one question, do not spend too much time on it.

Exercises

1. Let a filter be defined by the difference equation between the input x and the output y:

$$y(n) = x(n) + 2\,x(n\text{-}1) + x(n\text{-}2)$$

1.1 Demonstrate without any computation that this filter is a linear phase one.
1.2 What is the phase response of this filter?
1.3 If x is a white noise with variance 1, what is the autocorrelation function of y?

2. A signal x with sampling frequency f_s is generated using the following AR model:

$$x(n) = -0.9\,x(n\text{-}1) + \varepsilon(n)$$

with ε being white noise with variance σ_ε.
2.1. Write the expression for the power spectral density of x in terms of these parameters.
2.2. Is the power spectral density higher in the low-frequency or high-frequency range?

3. Find all the parameters of an AR model of order p = 1 for a signal x with autocorrelation function values:

$$R_{xx}(0) = 3 \qquad R_{xx}(1) = 1$$

4. Six samples $x(0)$, $x(1)$, …, $x(5)$ of a signal are available. One wishes to estimate the coefficients a_1 and a_2 of a linear predictor of order p = 2 on this signal.
The matrix equation to be solved in a least-squares sense, i.e. $-\mathbf{X}\boldsymbol{a} + \boldsymbol{e} = \boldsymbol{c}$, with $\boldsymbol{a}=[a_1\ a_1]^{\mathsf{T}}$ and $\boldsymbol{e}$ the error vector, to estimate these coefficients using the covariance method is:

$$-\begin{bmatrix} x(1) & x(0) \\ x(2) & x(1) \\ x(3) & x(2) \\ x(4) & x(3) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} e(2) \\ e(3) \\ e(4) \\ e(5) \end{bmatrix} = \begin{bmatrix} x(2) \\ x(3) \\ x(4) \\ x(5) \end{bmatrix}$$

Modify this equation for the case of the autocorrelation method.

Questions

1. Describe the advantages / drawbacks of FIR and IIR filters.

2.
a) What is a linear phase filter?
b) What are the properties of the impulse response of a linear phase filter?
c) What is a zero-phase filter?

3. Using the Welch algorithm to estimate the power spectral density:
a) What is the influence of the length of the block?
b) What is the motivation of multiplying each block by a window before the DFT?

4. Using the measurement of mean blood pressure and cardiac inter-beat intervals:
a) What is a suitable sampling frequency to analyse the control of the autonomic nervous system?
b) What is the advantage of using the auto-correlation method to estimate the power spectral density compared to discrete Fourier transform?

5. Suppose a microphone is recording someone speaking for some time ("period A") ; then the person stops talking and the microphone records only silence, which we can consider to be dominated by white noise, for some time ("period B"). An $AR(p)$ model, with $p > 0$, is fit to period A and to period B separately. In which period is the ratio *excitation variance / signal variance* the highest? Why?

6. When an AR model is estimated with an order that is too low, what can we expect to find about the estimated excitation signal ε? Why?

7. Regarding the estimation of the power spectral density based on AR or ARMA models:

a) What are the potential advantages with respect to non-parametric estimation approaches (e.g., Welsh method)?
b) What are the risks when the model order p is too low?

8. Regarding the Pisarenko harmonic retrieval method:

a) What type of signals is this method adequate for?
b) What model parameter(s) can the method estimate?
c) What model parameter(s) can the method not estimate?

9. Which window function should you choose for your spectrogram if you wanted to maximize your chances to differentiate between the heart rate and a motion frequency component that is very close to it? Why?

10. When estimating the sympathovagal balance, why would you probably have a lower time resolution on the low frequency component if you used the wavelet transform instead of the short-term Fourier transform?

11. One wants to estimate the instantaneous frequency (IF) of atrial fibrillation from the surface electrocardiogram (ECG). Following the subtraction of the ventricular activity (the atrial and ventricular activities overlap on the surface ECG), the remaining atrial signals are usually bandpass filtered to remove the spectral frequency components outside of the frequency range of atrial fibrillation.

a) From a theoretical perspective, why should the signals be bandpass filtered before estimating the instantaneous frequency?
b) What would be the benefit of bandpass filtering in terms of accuracy of IF estimation?

12.

a) How is defined the analytic signal?
b) An oscillation signal with slowly-time varying amplitude and frequency can be represented $x(t) = A(t)\cos\big(\Phi(t)\big)$. How can one define the instantaneous amplitude and frequency through the analytical representation of x(t)?